Magnetic Levitation Control With No Sensor

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1 Introduction

In this project we tried to control the height of a magnet being held up by an electromagnetic coil.

2 Circuit and System

2.1 Current Driver

The current driver circuit in this project is the same as we have used in previous motor control projects. In figure 1 the M is now the electromagnetic coil. The input voltage is $R_m i_m$, proportional to the current across the coil. a is the ratio of resistance on the $10k\Omega$ variable resistor. β is equal to $\frac{1}{\alpha}$. By matching the coil's resistance with an R_m of 90.9 Ω we set the required β to around 0.5.

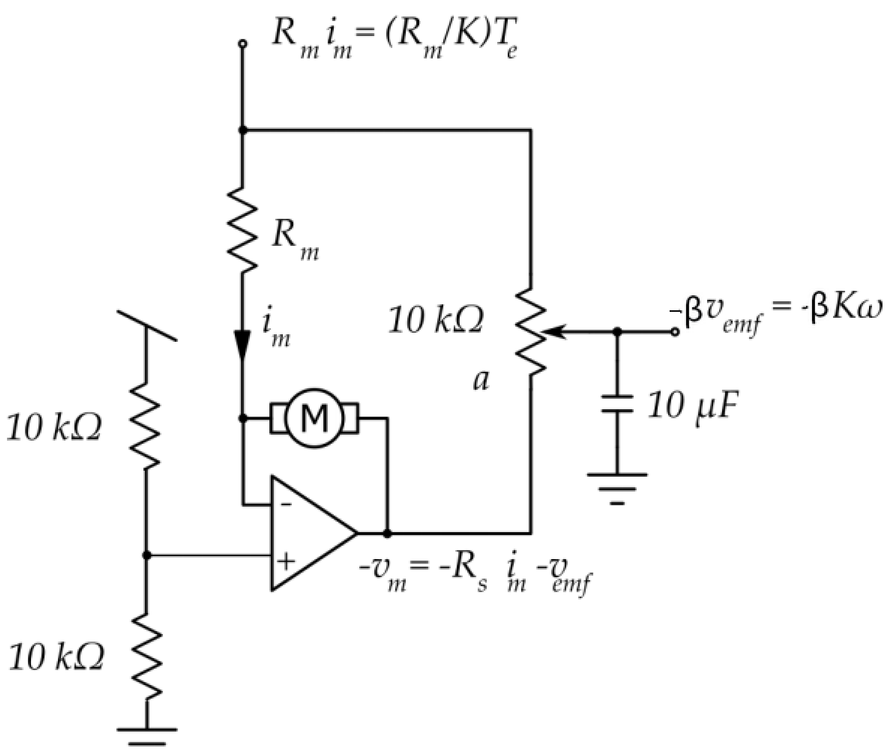

Figure 1: The circuit used to drive the coil.

2.2 Coil-Magnet System Characterization

From the paper "Axial Force Between a Thick Coil and a Cylindrical Permanent Magnet: Optimizing the Geometry of an Electromagnetic Actuator" by Will Robertson, Ben Cazzolato and Anthony Zander (pub. on IEEE in Sept, 2012), we got the following equation:

$$F_f(r_1, r_2, z) = \mu_0 I_1 I_2 z \sqrt{\frac{m}{4r_1 r_2}} \times [K(m) - \frac{m/2-1}{m-1}E(m)]$$

where I_1 and I_2 are the current in the coil and the equivalent current in the magnet, r_1 and r_2 are the coil radius and magnet radius, and z is the axial distance between their centers. $E(m)$ and $K(m)$ are the first and second elliptic integrals with parameter m.

This model applies to two single loops with currents passing through them. We modeled the magnet as a single loop with the magnet's radius and an amount of current that would generate a B field of equal strength to the magnet's. We modeled the coil as multiple loops with a radius halfway between the inner and outer radii of the coil. Rather than accounting for the position of each of the approximately 3200 wraps of wire on our actual coil we experimentally determined the distance at which the force on the magnet was equal to its weight, and used that to determine a coefficient for the coil.

We use this to produce a distance-force curve for a given current through the coil, as seen in figure 2.

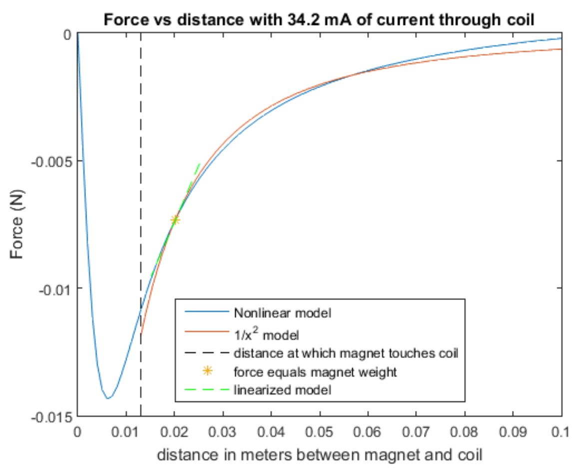

Figure 2: The force between the magnet and the coil

At distance = 0, measured between the centers of the magnet and the coil, the magnet is in the center of the coil, at a stable equilibrium. Any disturbance generates a force that returns the magnet to the center of the coil. However, this is not the equilibrium we are interested in. Our coil does not have space in the core, so the closest the magnet can get is approximately 1.3 cm. In this region of the graph, an increase in distance results in a decrease in restoring force pulling the two together. These can be related by

$$F_e \approx K\frac{i}{z^2}$$

where K is just a proportional constant containing the non-changing terms. We can linearize F_e around the equilibrium point z_{eq} and i_{eq}, where z_{eq} is the distance at which the lifting force (negative) is equal to the gravitational force on the magnet (positive) at a given current i_{eq}:

$$F_e \approx K\frac{i_{eq}}{z_{eq}^2} + \frac{\delta F_e}{\delta i}\Delta i - \frac{\delta F_e}{\delta z}\Delta z$$

$$F_e \approx z\frac{i_{eq}}{z_{eq}^2} + \frac{K}{z_{eq}^2}\Delta i - \frac{2Ki_{eq}}{z_{eq}^3}\Delta z$$

At a given equilibrium point, the equations of motion of the magnet can be modeled as following:

$$
\begin{aligned}
m(\ddot{z}) &= F_e + mg \\
m(\ddot{z_{eq} + \Delta z}) &= K\frac{i_{eq}}{z_{eq}^2} + mg + \frac{K}{z_{eq}^2}\Delta i - \frac{2Ki_{eq}}{z_{eq}^3}\Delta z \\
m(\ddot{\Delta z}) &= \frac{K}{z_{eq}^2}\Delta i - \frac{2Ki_{eq}}{z_{eq}^3}\Delta z
\end{aligned}
$$

The behavior around the equilibrium point with a constant current is that of a mass on a spring with a negative spring constant equal to $\frac{2Ki_{eq}}{z_{eq}^3}$; the further the mass strays from equilibrium, the more force pushes it even further away.

If we let u equal the speed, $u = \dot{\Delta z}$, and

$$
m\dot{u} + \frac{2Ki_{eq}}{z_{eq}^3}\int u dt = \frac{K}{z_{eq}^3}\Delta i
$$

Because magnetic force on the magnet is proportional to the current through the coil in this model, we can use this to construct a gyrator model that combines the electrical components with the mechanical components, allowing us to get a transfer function.

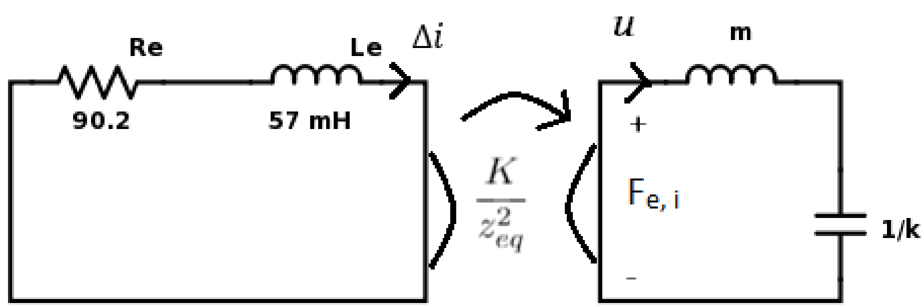

Figure 3: A gyrator model of the magnet-coil system around an equilibrium point.

Magnet velocity u is the 'current' flowing through the mechanical side of the gyrator, and F_{ei}, the component of the linearized force between the magnet and coil proportional to current, is the 'voltage'. It is related to the current offset, Δi from the equilibrium current, i_{eq} as follows:

$$
F_{e,i} = \frac{K}{z_{eq}^2}\Delta i
$$

The mass of the magnet m is analogous to an inductor. It acts as a memory component for which force (voltage) is proportional to the derivative of speed (current).

The spring effect mentioned earlier is also a memory component. Though there is no physical spring, the force does vary in proportion to the magnet's distance from the coil, $\int udt$, making it a capacitor. The value of the capacitor $\frac{1}{k}$ can be derived from:

$$
k = \frac{2Ki_{eq}}{z_{eq}^3}
$$

Note that this is a negative spring because as the magnet move closer to the coil from the equilibrium point, the attractive force increases rather than decreases, and vice versa. Thus, the value for k should be negative.

When these components are reflected across the gyrator as electrical components, the effective circuit becomes

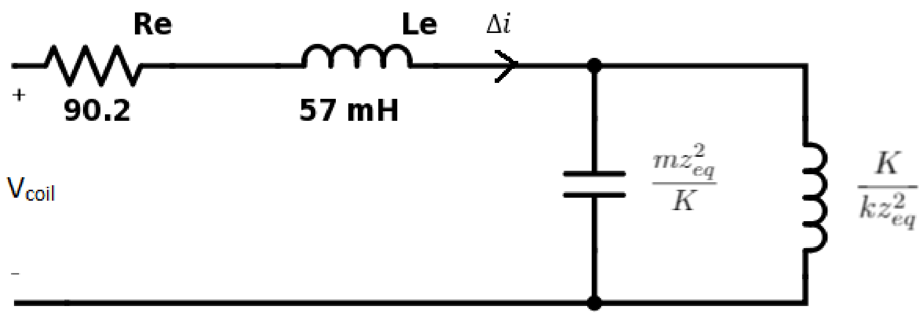

Figure 4: Model of the magnet-coil system, mapping the mechanical components into electrical components.

2.3 Transfer Function of system

Let $L = \frac{K}{kz_{eq}^2}$ and $C = \frac{mz_{eq}^2}{K}$

The transfer function is then

$$\frac{V_{coil}}{\Delta I} = \frac{L_e LCs^2 + L_e LCs^3 + (L_e + L)s + R_e}{LCs^2 + 1}$$

Note that $LC = \frac{m}{k}$. The poles are then

$$s = \pm\sqrt{\frac{k}{m}},$$

since k is negative.

3 Measuring the poles of the system

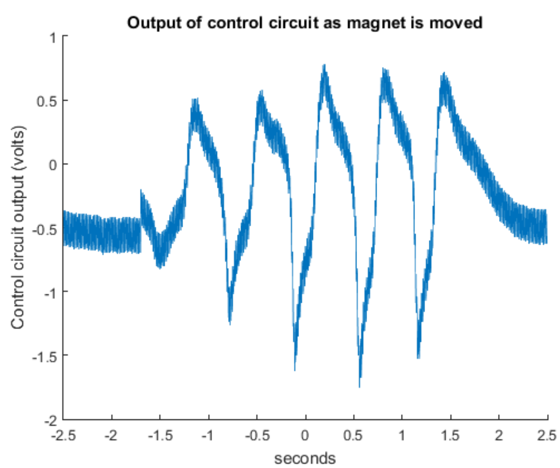

Figure 5: The control circuit responds as the magnet is moved back and forth by hand.

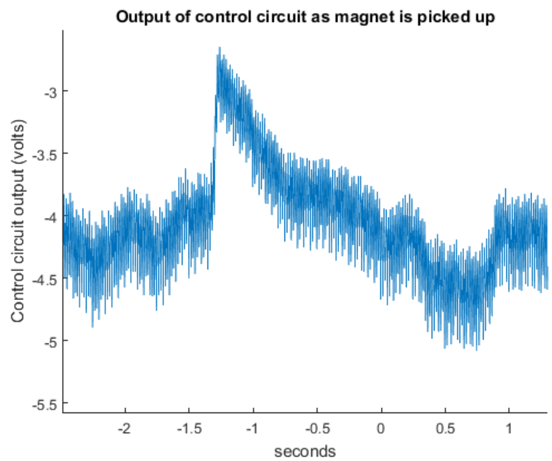

Figure 6: At -1.3 seconds, the magnet passes equilibrium and is picked up by the coil. It sticks to the coil, and the slope from -1.2 to +1 seconds is the integrator drifting back.

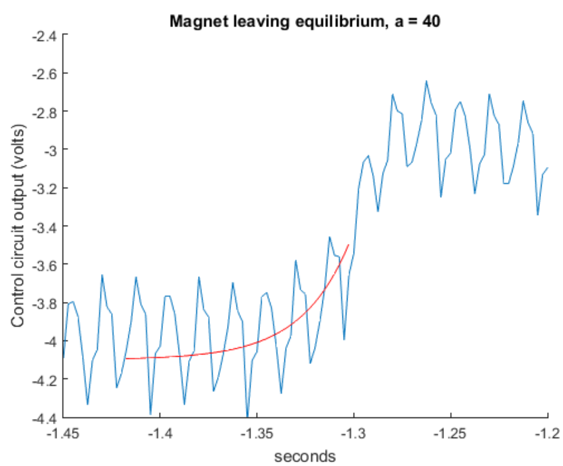

Figure 7: Model of the magnet-coil system, mapping the mechanical components into electrical components.

The graph above shows an exponential fit for the voltage response to the acceleration of the magnet as it leaves equilibrium with the equation

$$v(t) = 1e^{40t}$$

but the data is too noisy for us to consider this accurate.

4 Controller circuit and transfer functions

The overall controller diagram is as follows:

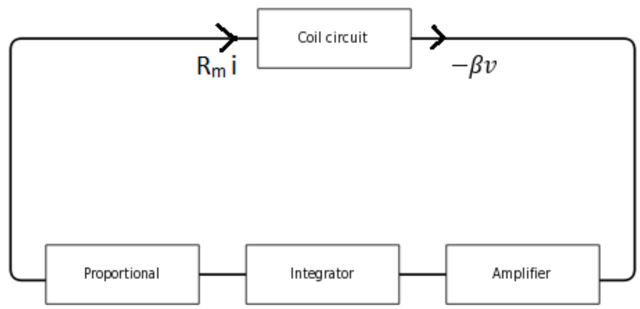

Figure 8: The Overall controller diagram

The individual components are explained below.

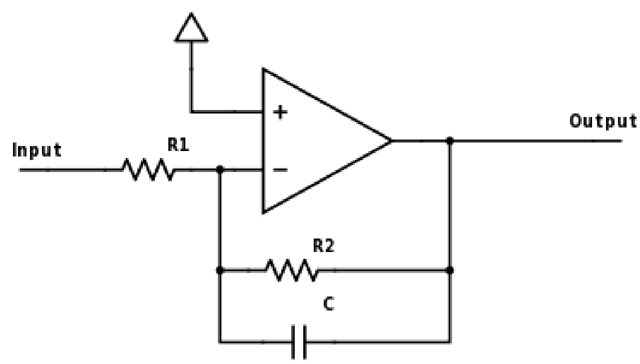

Figure 9: The amplifier block

The transfer function for this amplifier is

$$\frac{V_out}{V_{in}} = -\frac{R_2 C}{R_1(R_2 + C)}$$

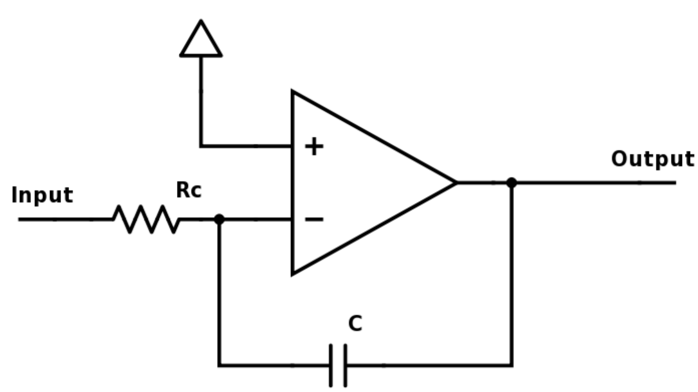

Figure 10: The integrator block

The transfer function for this integrator is

$$\frac{V_out}{V_{in}} = \frac{1}{R_cCs}$$

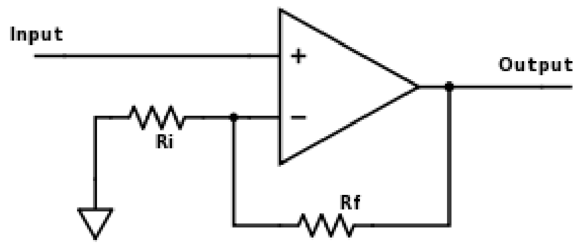

Figure 11: The proportional block

The transfer function for this proportional gain is

$$\frac{V_out}{V_{in}} = \frac{R_f}{R_i}$$

5 Results

We weren't able to successfuly levetate our magnet because the induced current from the magnet moving was too small for us to use in our controller. In the data shown in figure 5 on page 4 and figure 6 on page 5 you can see that the signal's noise had about the same magnitude as the largest disturbances from which the system could possibly recover. In addition, when we tried to amplify these noisy signals enough to actually levitate the magnet, we encountered problems with op-amps quickly reaching their limits, and small errors in the set point of β causing the system to enter into a self correcting control loop which changed the control value in response to the set point but not in response to the magnet's motion.

When we set up the circuit with less amplification, we were able to observe the system lifting the magnet more slowly than an uncontrolled magnet, and we could see the commanded current decrease as the magnet accelerated up, however, the magnet still jumps up to the coil rather than floating. The videos below show the coil lifting the magnet and the control circuit responding to the magnet's motion.

5.1 Videos

magnet lifting
conrol system responding to magnet motion