Inverted Pendulum Using Antigravity Motor Control

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1 Introduction

In this project we tried to control a pendulum attached to a lego motor to move towards the upright position without using any sensors such as an encoder. We did this by measuring the voltage across the motor, feeding that through a controller and commanding the motor current with the controller's output.

We started with an integrator controller which we used in previous labs to make a position controller. We adjusted values in our integrator and amplifier so that we could sense the position of the pendulum through multiple rotations. Then we explored the behavior which we saw in a class demo where a position controller with a setpoint of zero and gains which put the system near instability exhibits an inverted pendulum behavior. We then looked at the system and tried to explain what might cause this inverted pendulum behavior.

2 Circuit and System

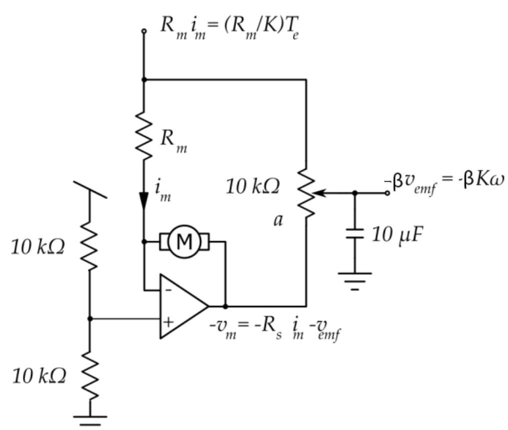

Figure 1: We are using the same motor driver as in previous circuits. The input voltage is $R_m i_m$, proportional to the current across the motor. a is the ratio of resistance on the $10k\Omega$ variable resistor.

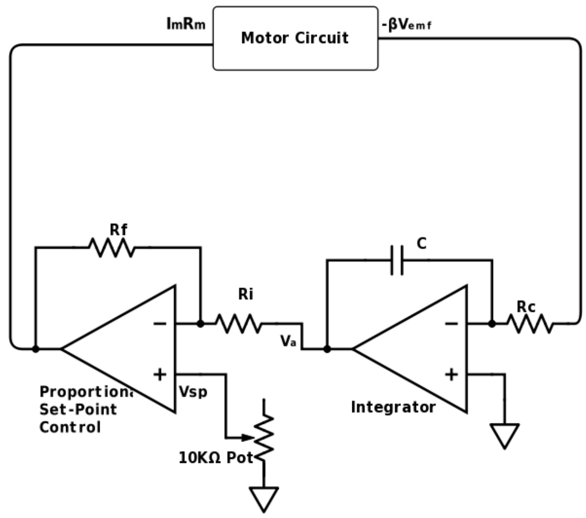

Figure 2: The control circuit includes an integrator and a potentiometer-adjusted proportional controller.

3 Transfer Functions

3.1 Motor

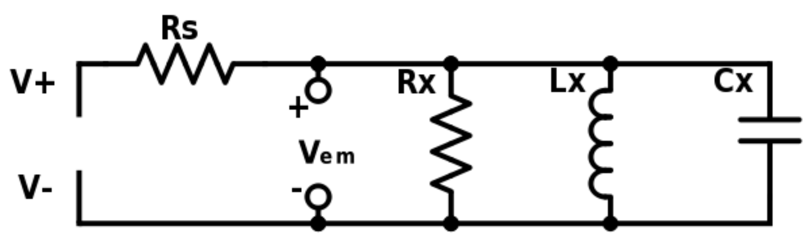

Figure 3: A model of the motor as a parallel RLC circuit. Rx, Lx, Cx are mechanical components. The torque $\tau = K_m i$ where i is the current through the motor. The electromotive force $v_{emf} = K_m \omega$ where ω is the angular velocity of the motor.

In previous labs we characterized this motor without a pendulum ($L_x = 0$) and found the following values. I is the moment of inertia of the motor and b is the mechanical damping factor. R_s is the electrical resistance of the motor, i is the current across the motor, and K_m is the motor constant.

$$R_s = 33.3\Omega$$
$$K_m = 0.66Vs$$
$$I = 0.0156\frac{N}{m/s}$$
$$b = 0.01Nm$$

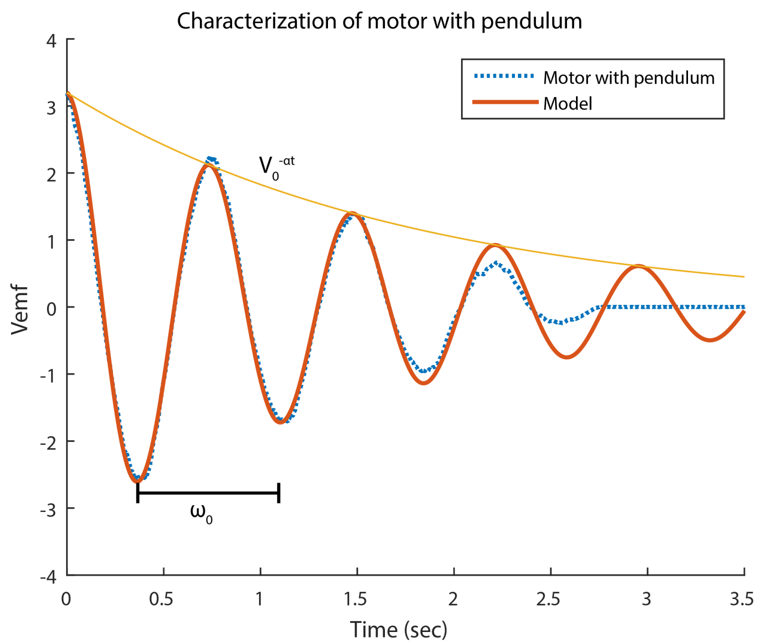

Figure 4: We attribute the nonlinear decrease in amplitude at low speeds to be caused by friction.

From this model we can find

$$\alpha = \frac{1}{2R_x C_x} = 0.56$$
$$\omega_0 = \sqrt{\frac{1}{L_x C_x}} = 8.49\text{rad/sec}$$

which allow us to calculate

$$R_x = \frac{{K_m}^2}{b} = 43.56\Omega$$
$$L_x = \frac{2{K_m}^2}{lmg} = 0.3384\text{H}$$
$$C_x = \frac{I}{{K_m}^2} = 0.041\text{F}$$

Note that L_x is introduced by the pendulum and l is the length of the pendulum, m its mass, and $g = 9.8ms^{-2}$. The transfer function of the motor is now

$$\frac{V_{emf}}{i_m} = \frac{\frac{s}{C_x}}{s^2 + \frac{1}{R_x C_x}s + \frac{1}{L_x C_x}}$$

$$Q = \frac{\omega_0}{2\alpha} = 7.56$$

3.2 Integrator

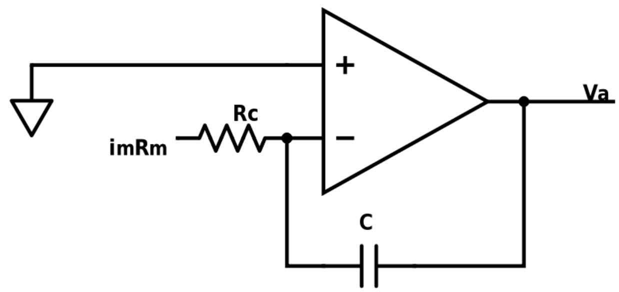

Figure 5: The integrator block

The transfer function for this integrator is

$$\frac{V_a}{V_{in}} = \frac{1}{R_c C s}$$

3.3 Proportional

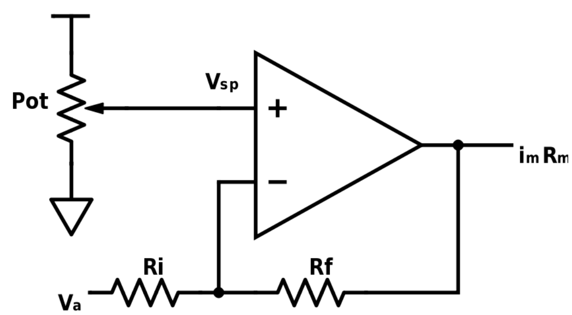

Figure 6: The integrator block

The inputs and outputs of this proportional controller are related by

$$\frac{V_{out} - V_{sp}}{R_f} = \frac{V_{sp} - V_a}{R_i}$$

One way to solve for a transfer function is to set V_{sp} to 0, yielding a simple proportional gain:

$$\frac{V_{out}}{V_a} = \frac{-R_f}{R_i}$$

Another is to model V_{out} as a gain K_p multiplied by the difference between V_{in} and V'_{sp}, a value derived from V_{sp}. This model has two inputs and therefore does not have a out/in transfer function ratio.

$$V_{out} = -K_p(V_a - V'_{sp})$$
$$V'_{sp} = \frac{R_i + R_f}{R_f} V_{sp}$$

3.4 Overall

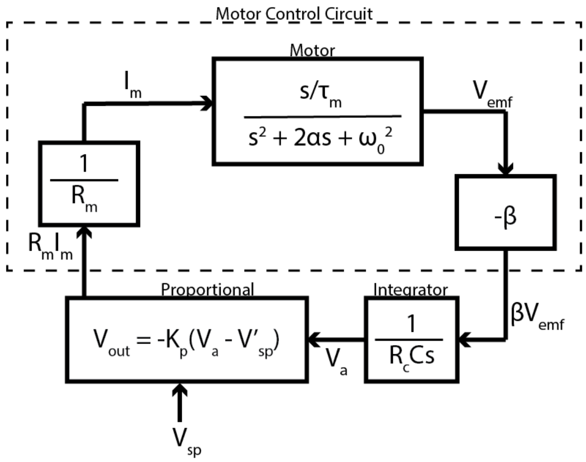

Figure 7: The overall system diagram

To characterize the entire system we must simplify the system diagram. For convenience, we will define $\frac{R+f}{R_i}$ as K_1 and R_cC as τ_i making the controller transfer function $\frac{-\beta K_p}{\tau_i s}$ when $V_{sp} = 0$.

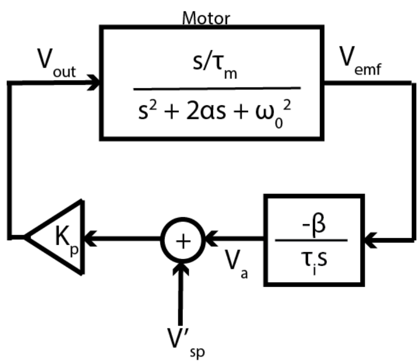

Figure 8: The simplified system diagram

A transfer function for this entire system is

$$\frac{V_{emf}}{V'_{sp}} = \frac{K_p \frac{s}{\tau_m}}{s^2 + 2\alpha s + \omega_0^2 + \frac{\beta K_p}{\tau_i \tau_m}}$$

We can further get a transfer function in terms of θ and V'_{sp} by multiplying both sides by $\frac{1}{s}$.

$$\theta = \omega \frac{1}{s} = \frac{V_{emf}}{K_m} \frac{1}{s}$$

$$\frac{\theta}{V'_{sp}} = \frac{\frac{K_p}{K_m \tau_m}}{s^2 + 2\alpha s + \omega_0^2 + \frac{\beta K_p}{\tau_i \tau_m}}$$

4 Incorporating Torque Feedback

In order to induce the inverted pendulum behavior from our circuit, we had to adjust the potentiometer in the original motor circuit which was previously set to cancel the torque portion of the voltage across the motor. We therefore changed the output of the motor circuit to include a term proportional to current.

$$-\beta V_{emf} \text{ is replaced with } ((1-\beta)R_m - \beta R_s)I_m - \beta V_{emf}$$
$$K_1 = (1-\beta)R_m - \beta R_s$$

Using the controller's transfer function, we can find:

$$V_{out} = \frac{-R_f}{R_i R_c C s}(K_1 I_m - \beta V_{emf})$$

based on our circuit:

$$V_{out} = I_m R_m$$

making the transfer function of the motor:

$$\frac{V_{emf}}{V_{out}} = \frac{\frac{s}{R_m C_x}}{s^2 + \frac{1}{R_x C_x}s + \frac{1}{L_x C_x}}$$

Using the V_{out} from above, we can find:

$$\frac{V_{emf}}{I_m} = \frac{\frac{\tau_i K_p K_1}{\tau_i K_p \beta + R_m C_x}s^2}{s^2 + \frac{R_m}{R_x(\tau_i K_p \beta + R_m C_x)}s + \frac{R_m}{L_x(\tau_i K_p \beta + R_m C_x)}}$$

Or, if we let $\tau_m = R_m C_x$, We can get the transfer function as

$$\frac{V_{emf}}{I_m} = \frac{\frac{\tau_i K_p K_1}{\tau_i K_p \beta + \tau_m}s^2}{s^2 + \frac{\tau_m 2\alpha}{\tau_i K_p \beta + \tau_m}s + \frac{\tau_m \omega_0^2}{\tau_i K_p \beta + \tau_m}}$$

Following the same logic as before, we can find the transfer function $\frac{\theta}{I_m}$:

$$\theta = \omega\frac{1}{s} = \frac{V_{emf}}{K_m}\frac{1}{s}$$
$$\frac{\theta}{I_m} = \frac{\frac{\tau_i K_p K_1}{K_m(\tau_i K_p \beta + \tau_m)}s}{s^2 + \frac{\tau_m 2\alpha}{\tau_i K_p \beta + \tau_m}s + \frac{\tau_m \omega_0^2}{\tau_i K_p \beta + \tau_m}}$$

The steady state of this system is zero, unlike before, where the steady state was related to the setpoint. This "zero position" is either straight up or straight down, where the required torque (and therefore current) is zero.

The transfer function also shows that the controller is affecting both 2α and ω_0^2. In both cases, the original values are multiplied by $\frac{\tau_m}{\tau_i K_p \beta + \tau_m}$. All of the values in this multiplier are defined as positive, therefore the controller theoretically cannot make the system unstable, however, it can decrease α so that it approaches instability.

5 Experiment

We changed the value of R_c and β through two 10k potentiometers. Changing β influence how close to instability the system is (instability means when α is negative). Changing R_c modified how fast the pendulum corrects to the upright position. If it corrects two fast, the pendulum tends to overshoot, but if it corrects too slowly, the pendulum can only be stable in a small range.

5.1 The Time It Worked

Below are the values we recorded from our working inverted pendulum:

$$\beta = 0.41$$
$$R_c = 5.25k\Omega$$
$$C = 100\mu F$$
$$R_f = 100K\Omega$$
$$R_i = 10K\Omega$$
$$V_{sp} = 0V$$

Based on these values the multiplier in the transfer function is:

$$\frac{\tau_m}{\tau_i K_p \beta + \tau_m} = 0.386$$

The resulting transfer function is

$$\begin{aligned}
\frac{V_{emf}}{I_m} &= \frac{\frac{\tau_i K_p K_1}{\tau_i K_p \beta + \tau_m} s^2}{s^2 + 0.386 * 2\alpha s + 0.386 * \omega_0^2} \\
&= \frac{\frac{\tau_i K_p K_1}{\tau_i K_p \beta + \tau_m} s^2}{s^2 + 0.386 * 2 * 0.56s + 0.386 * 8.49^2} \\
&= \frac{\frac{\tau_i K_p K_1}{\tau_i K_p \beta + \tau_m} s^2}{s^2 + 0.43s + 55.6}
\end{aligned}$$

And,

$$\alpha_1 = 0.22s^{-1}$$
$$\omega_0 = 5.27rad/s$$
$$Q = \frac{\omega_0}{2\alpha} = 12.2$$

	Original Motor Circuit	With Torque feedback
$\alpha(s^{-1})$	0.56	0.22
$\omega_0 rad/s$	8.49	5.27
Q	7.56	12.2

α is a lot closer to zero. Since if α is negative the system would be unstable, an α close to 0 means that it is closer to instablity. The actual α was much closer to 0 because the system is nonlinear. We measured the disturbance response of our inverted pendulum and found an average α of approximately 0.1, however, the damping increased as the pendulum approached its steady state, as seen in the graph below. In fact, with a large enough disturbance, the system would enter a steady state where it would have large oscillations which weren't decreasing at all.

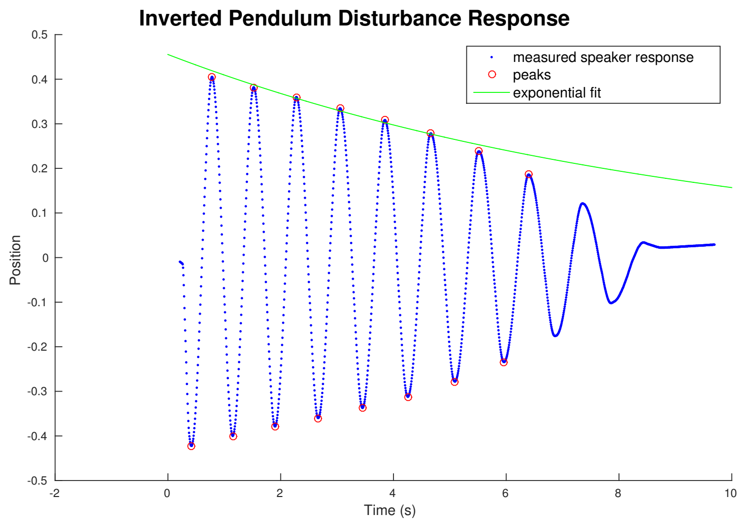

Figure 9: Inverted pendulum response, shows a lower α and a nonlinear response, as the amplitude of the oscillations does not decrease according to an exponential